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Shape optimization of a flat channel with an array of discrete, flush-mounted heat sources on one plate being cooled by forced convective water

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Abstract

Purpose – The purpose of this paper is to provide a suitable linkage between a computational fluid dynamics code and a shape optimization code for the analysis of heat/fluid flow in forced convection channels normally used in the cooling of electronic equipment.

Design/methodology/approach – A parallel-plate channel with a discrete array of five heat sources embedded in one plate with the other plate insulated constitutes the starting model. Using water as the coolant medium, the objective is to optimize the shape of the channel employing a computerized design loop. The two-part optimization problem is constrained to allow only the unheated plate to deform, while maintaining the same inlet shape and observing a maximum pressure drop constraint.

Findings – First, the results for the linearly deformed unheated plate show significant decrease in the plate temperatures of the heated plate, with the maximum plate temperature occurring slightly upstream of the outlet. Second, when the unheated plate is allowed to deform nonlinearly, a parabolic-like shaped plate is achieved where the maximum plate temperature is further reduced, with a corresponding intensification in the local heat transfer coefficient. The effectiveness of the computerized design loop is demonstrated in complete detail.

Originality/value – This article offers a simple, harmonious technique for optimizing the shape of forced convection channels subjected to pre-set design constraints.

Keywords Optimum design, Laminar flow, Convection, Cooling, Flow, Fluid dynamics

Paper type Research paper

Nomenclature

- b initial channel width, heater width, heater spacing
- D_h hydraulic diameter (2b)
- g gravitational acceleration
- h mean heat transfer coefficient
- k thermal conductivity

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Nu mean Nusselt Number (hb/k)
p pressure
Pr Prandtl number (ν/α)
q_w heater heat flux
Re Reynolds number ($2bV/\nu$)
T temperature
u, v x- and y-velocity components
V mean velocity
x, y Cartesian coordinates

Greek symbols
α thermal diffusivity
ν kinematic viscosity
ρ density

Subscripts
o inlet value
w plate value

Introduction

Much of the current research and development on forced convective heat transfer is focused on finding and/or exploring means of improving traditional approaches that have an impact on cooling/heating applications in engineering. Such techniques include using finned surfaces, acoustical, electrical or hydrodynamic perturbations, pulsation and others (Bergles, 1998; Manglik, 2003). Usually, these enhancement schemes come either with added manufacturing or energy costs or restrictions in their potential application due to their particular natures.

A detailed review of the archival literature on fluid dynamics and heat transfer reveals that the optimal shape of forced convective flat channels has not been investigated so far (Bar-Cohen *et al.*, 2003), and of course its physical features are not understood. In the present study, we examine the heat transfer enhancement that can be derived from a simple method of changing the shape of a forced convective flat channel. We will, in fact, focus the present discussion on the sequential design process that encompasses shape changes and optimized configurations, as it is the overall process that is the most interesting aspect of the study, rather than the quantitative improvement in the heat transfer coefficient. Indeed, the present methodology can be used to find the optimal design based on the desired objective (for example, maximize heat transport, minimize pressure losses, etc.) and the necessary constraints dictated by a specific application. The optimal design will be responsive to the different constraints and the multiple applications. Finally, the design methodology to be implemented often serves to illuminate flaws in the proposed design or model by moving toward an unrealistic or undesirable design.

Heat generation in individual electronic chips is continually increasing in modern electronic equipment as a result of increasing circuit integration and power (Bar-Cohen *et al.*, 2003). Despite that a variety of thermal control techniques have evolved in the past to meet heat dissipation requirements, new alternatives continue to be sought to satisfy the demands. Thermal control techniques are needed that enhance the relatively low heat transfer coefficients that are typically found in cooling channels for electronic equipment.

The objectives of the present study are to demonstrate the process of finding optimal designs for the flat channels used to cool a discrete array of heat sources such as may be found in electronic equipment. The designs will be dependent on pre-set constraints and various shape deformation variables. As expected, a prominent application of the optimized flat channels arises in the miniaturization of electronic packaging that is subject to strict space and/or weight constraints (Bar-Cohen *et al.*, 2003).

It is well known from fluid physics that any increment in fluid velocity can lead to an increase in the heat transfer coefficient (Kays and Crawford, 1993). A potential technique to increase heat transfer coefficients in cooling channels is the use of converging plates instead of parallel plates. Such converging channels accelerate the coolant, producing increasing enhancement to heat transfer coefficients as the channel narrows downstream. The heat transfer enhancement is particularly needed in downstream locations where heat transfer coefficients are lower for constant-width channels.

The flat channel configuration is relevant in cooling applications of electronic equipment and heat exchangers. Research in the past has been focused on the fundamental question of how to distribute a number of discrete heaters in a flat channel involving forced convection or natural convection. The former problem has been addressed in Incropera *et al.* (1986), Mahaney *et al.* (1990), McIntire and Webb (1990), Shaw and Chen (1991), Wang (1992), Joshi and Rahall (1994), Morega and Bejan (1994), Gupta and Jaluria (1998), Chen *et al.* (2001), Furukawa and Yang (2003), da Silva *et al.* (2006), Gonzalez *et al.* (2007), Cheng *et al.* (2008) and Kou *et al.* (2008), whereas the latter problem has been addressed by Morrone *et al.* (1997).

Only two experimental works have been reported on forced convection in parallel-plate channels with one plate incrusted with discrete isoflux heaters and the other plate insulated. McIntire and Webb (1990) undertook experiments to measure the heat transfer characteristics of an array of four discrete heat sources embedded in flush configurations. Airflow rates yielding Reynolds numbers (based on hydraulic diameter) of 1,000 and slightly higher were employed. Joshi and Rahall (1994) investigated experimentally mixed convection from a column of 10 heat sources exposed to forced water for Reynolds numbers (based on the hydraulic diameter) up to 1,700. In the two studies, the heater temperature increased in the flow direction as a double-value function of the power level and Re value, as expected.

Concerning non-parallel-plate channels, Su and Lin (1991) performed a numerical computation of laminar forced convection in convergent and divergent channels with both plates at equal temperatures. For the specific case of convergent flows, the results of these authors showed that the pressure decreases while the convective heat transfer increases with accretions in the taper angle.

Model

The initial model is a two-dimensional parallel-plate channel with a row of five flush-mounted heat sources embedded along the left plate, while the right plate is thermally insulated. Figure 1 sketches the geometry for the initial model. The coolant is water. The channel width b is 1 m and will remain fixed at 1 m at the channel entrance, but will be allowed to vary downstream therefrom. The hydraulic diameter D_H of the channel is equal to two times the channel width b. The heaters are 1 m wide and are spaced 1 m apart. The unheated entry length and the inter-spaces between the heaters are thermally insulated. The water flows upward with an initial unheated length of 7.5 D_H for flow development. The no-slip condition is enforced at the plates and uniform velocity and uniform temperature profiles are specified at the inlet. The governing equations for two-dimensional, incompressible, constant property fluid in Cartesian coordinates are:

Mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

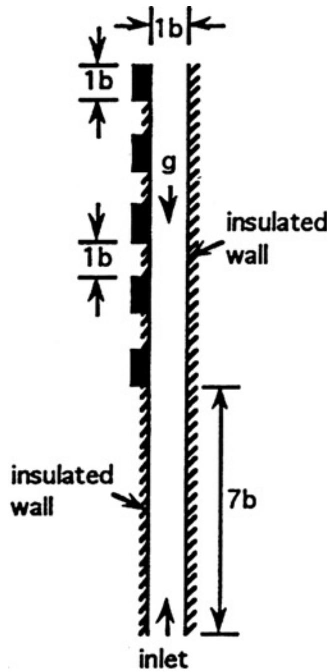


Figure 1.
Sketch of the initial channel with an array of flush-mounted heaters

x-momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}\right)+\nu\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

y-momentum:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}\right)+\nu\left(\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}\right)+\alpha\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

For simplicity, it is assumed that water properties remain constant during the course of the water flow in the channel.

Computational procedure

For the initial model we choose a laminar Reynolds number of 1,000 based on the mean velocity V and the hydraulic diameter $D_h = 2b$, guaranteeing that the water flow is laminar in the entire channel. Further, although we changed the width b and mean velocity V as a function of the streamwise direction y, the Reynolds number remains constant because the mass flow rate $\dot{m} = \rho V b$ was held constant. The heat transfer coefficient is defined as $h = q_w/(T_w - T_o)$, where q_w is the specified heater heat flux, T_w is the plate temperature and T_o is the inlet fluid temperature. The Nusselt number is defined as $Nu = hb/k$, where b is the channel width. The inlet ambient temperature is set at 20 °C, and the water properties are determined at this temperature giving a Prandtl number $Pr = 7.56$. The goal is to compute the local plate temperatures and the heat transfer coefficients, and use these data for comparisons between consecutive designs.

In the next section, the position/shape of the right plate will be allowed to change, except for its location at the inlet. Actually, this step will enable us to optimize the channel design as to minimize the local plate temperatures or maximize the heat transfer coefficients. The system of conservation equations (1)-(4), subject to the boundary conditions was solved by the finite-volume method described by Patankar (1980). After the sensitivity analysis of the computational mesh has been completed, the numerical solution delivers the velocity fields u(x, y), v(x, y) and the temperature field T(x, y) of the water. The commercial computational fluid dynamic (CFD) code FLUENT[1] was used for the first phase of the numerical calculations.

Hwang and Fan (1964) computed the axial variation of the Nusselt number in the developing flow of various Prandtl number fluids inside parallel-plate channels with constant heat flux boundary condition. The data repeated in Table 37 of Shah and London (1978) has been used for validating the outcome of the computer code.

Optimization methodology

Optimization science is a mature science developed by applied mathematicians and engineers for several decades (Thévenin and Janiga, 2008). This book, along with recent publications by Gosselin *et al.* (2009) and Hilbert *et al.* (2006) contain references devoted to shape optimization of channels using modern evolutionary algorithms, such as genetic algorithms.

Computerized engineering analysis tools such as CFD codes and stress analysis codes have also become mature tools over the past two or three decades in the engineering community, including the industrial/commercial sector. What has not matured is the synergistic combination of computerized engineering analysis tools and computerized optimization tools. As this happens, engineers will not only have the ability to analyze designs of existing and new system components, but also to actually start with an initial design. Then, allowing the number "n" of design variables that define the design to vary, engineers find the best design that optimizes the desired objective which exists somewhere in the n-dimensional design space. The design space, of course, is bounded by constraints that are determined by the design engineer or by the engineering problem and will in general be different for each application. The optimizer employs a search algorithm to find design extrema in the design space. In the present study, the algorithm used is the gradient-based hybrid sequential-quadratic-programming generalized-reduced-gradient method of Parkinson and Wilson (1988).

The behavior of many system components is a function of their intrinsic geometries. The ability to deform the shapes of such components to arbitrary shapes that are not found in the stable of standard shapes (plates, cylinders, spheres, etc.) enhances the design potential of the combined optimizer and analyzer software. This is based on the assumption that the analyzer can perform the analysis on a modified geometry. The present optimization problem employs an effective arbitrary shape deformation algorithm to change the shape of the initial geometry of the component to be optimized. The arbitrary shape deformer (ASD) is a three-dimensional function that can use either Bezier functions or non-uniform rational B-splines (NURBS). The ASD acts as an intermediate processor on the geometry of the model. The geometry can be defined as computer-aided design data, free-form entities (e.g. Bezier curves, NURBS, etc.) or mesh entities (finite element, finite volume, etc.). The ASD is user-defined to embrace the volume containing the geometry model to be optimized. As the control points of the ASD are moved, the model geometry is deformed. The updated model's geometry is then used for the next analysis call. The shape deformation algorithm allows from fine local to gross global shape deformations using very few shape-related design variables to achieve the desired changes in shape. The major component of the design loop needed for automatic shape optimization is the engineering analysis software. Such software can be any computer-based analysis code including commercial numerical simulation software. The present study employed the commercial finite-volume CFD code FLUENT.

The methodology for achieving an optimal design is explained later. With the design tools discussed earlier made available, the design engineer must still implement them effectively to achieve the desired result. After identifying the problem to be solved, a proposed solution may be synthesized to address the problem. This step is usually accomplished through proposing a design already familiar to the engineer that may require some modification for application to the new problem. The next step can be to determine if the proposed design is a feasible design either by performing an analysis or by building a prototype and testing it. If the design is not feasible or is not good enough, the engineer may use his own judgment to make alterations until a better design is found. This part of the process can be very time consuming, especially if the proving is done experimentally. In fact, many numerical analysis coding efforts over the years have been justified by promising great improvements in the proving cycle by being able to eliminate many designs as infeasible through performing computer simulations to show such. Even though the computer simulation has proved worthy many times, it still may require some experimental validation; this is especially true for CFD codes in large part. While numerical simulation codes have indeed accelerated the design cycle, the next step is to connect such codes into an automatic design loop where the analysis code is used to determine the performance of the initial design. Thereafter, an optimizer is used to suggest modifications of the specified design variable to improve the design. Hence, upon proposing an initial engineering model, the next step in the design process for the present study was to pose the optimization problem. Figure 2 delineates a flowchart containing the major elements in the design optimization cycle.

The optimization problem is postulated by first identifying all of the variables that exist for the problem. Those variables that are to be allowed to change are then chosen from this list and called design variables. The design function or functions that are to be optimized are then identified and constructed. The function to be optimized is called the objective function and is a function of all the problem variables. There will be constraints placed on some of the design functions and these are essential to a well-posed problem. In fact, if a constraint is forgotten, the optimizer will exploit that fact and try to find an optimum that may lie outside true feasible design space. Hence, the optimization process may help determine the well-posedness of the model.

Upon specifying the design variables (including the shape variables), the objective function(s) and the constraints, the optimizer is turned "on" and takes control of the process. The optimizer will call the analyzer to obtain the performance of the initial model. Thereafter, it will then determine how the model performance is affected when each of the design variables is perturbed. This is sometimes called a sensitivity analysis and produces gradients that will be used to determine the search direction.

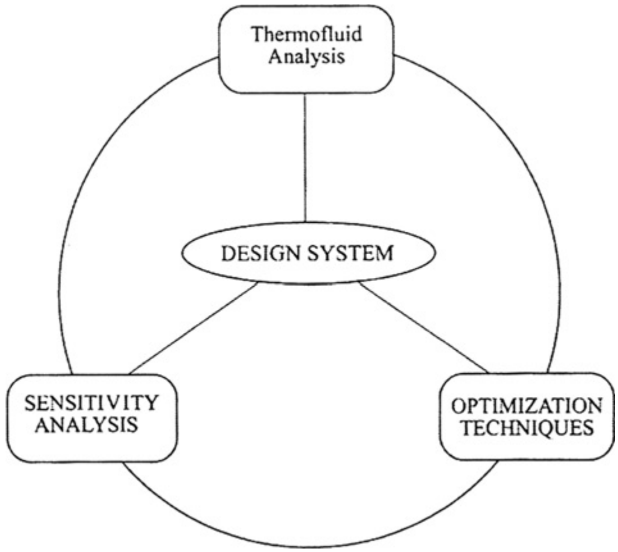


Figure 2.
The design optimization cycle

The sensitivity analysis includes perturbing the shape of the model. Having found a search direction from the sensitivity analysis, the optimizer will then modify the design variables accordingly to pursue the search. Upon finding an optimum (minimum or maximum) to a given tolerance, the optimizer quits. Of course, the optimal design is responsive to the accuracy of the analyzer and the degree it is correctly employed by the engineer. Furthermore, it may have found a local extremum, rather than the global extremum. The engineer can perform some analyses to help determine whether the extremum is the global one and can use the optimizer to map the design space or try a different initial model and see where the optimization leads. Or the engineer may relax or further restrict the constraints to see what happens. Even if the design is not the global optimum, the design is feasible based on the posed problem and may well be good enough for the intended application. For the second part of the study, the shape optimization code Sculptor[2] is utilized. The latter code possesses several robust and efficient gradient-based and non-gradient-based optimization search routines.

Details of the intermediate designs that are conducive to the optimal design of the cooling channel under study are given in the next section.

Numerical results

The initial model chosen corresponds to the flat channel with two parallel plates sketched in Figure 1a. The design variables identified for the model problem include the fluid, the plate temperatures, the shape of the unheated plate, which requires only one shape-related design variable, and the local velocity. The list of constraints applied to the optimization problem included the following:

- constant mean inlet velocity related to $Re = 1{,}000$;
- constant water properties at 20 °C;
- constant heater heat flux $q_w = 1{,}050\ \mathrm{W/m^2}$;
- straight shape for the heated plate;

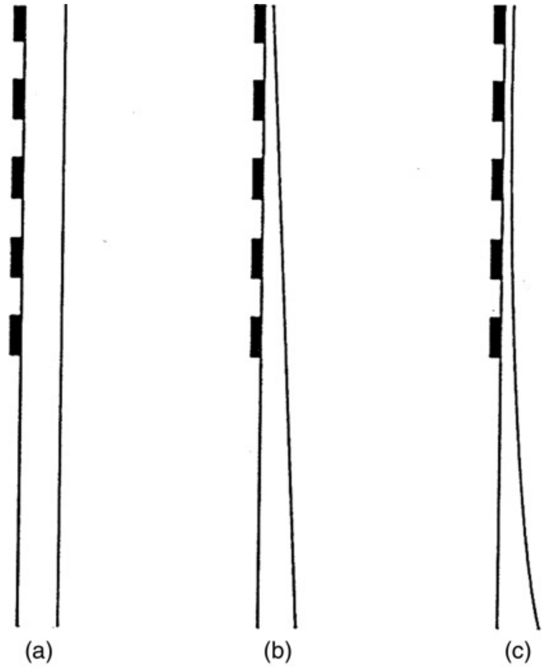
- uniform inlet temperature $T_0 = 20\,^{\circ}C$;
- constant inlet width $b = 1$ cm; and
- linear shape for the insulated plate.

In choosing the objective function, we could not maximize the actual heat transfer rate because it is fixed, as one would expect for the realistic application. Therefore, the objective function chosen for the first design optimization was to minimize the temperature of the fluid adjacent to a discrete heater at the furthest downstream point. Based on physical grounds, this should be the point of maximum fluid temperature for the initial model. Upon executing the optimization problem as stated above, we discovered two flaws in the optimization problem we had posed. The first problem encountered was that the model problem was under-constrained. That is, the optimizer found that it could bring the top of the unheated plate closer and closer to the heated plate, until the outlet was unrealistically narrow and the accompanying pressure drop undesirably high. The second problem we encountered was that the performance of the cooling channel changed such that the fluid temperature adjacent to a heater at the furthest downstream point was no longer the maximum temperature. These factors are indicative that the objective function was poorly posed. To remediate the ill-posed situation, we chose a maximum pressure drop (300 a) from the channel inlet to the channel outlet (based on transverse pressure averages) as a further constraint and reposed the objective function as the minimization of the sum of all the nodal temperatures along the heated plate. We left unchanged the constraint to keep the shape of the unheated plate linear. Knowing now that the optimization problem is well-posed now, we allowed the optimizer to search for an optimum. The optimum was found, not surprisingly, at the condition of maximum allowable pressure drop.

The final optimization problem posed was the same as the previous one except that the unheated plate was allowed to change shape in a nonlinear fashion. This required only two shape-related design variables and allowed the optimizer in conjunction with the shape deformer to find the best arbitrary shape that minimized the objective function. Upon execution of the optimization problem, the optimal shape discovered for the unheated plate was found to have a parabolic-like profile. Again, the constraint of a maximum pressure drop proved to be an active constraint. That is, the optimum occurs on the curve in design space that represents the maximum pressure drop. Figure 3 illustrates the shapes for the three channels discussed:

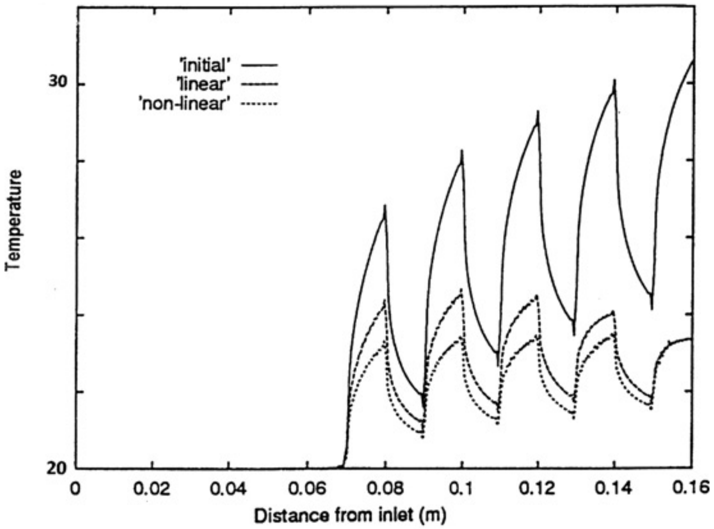
- the initial channel design with two parallel plates;
- the channel shape with the unheated plate constrained to be linear; and
- the channel shape where the unheated plate was allowed to have a nonlinear shape.

Figure 4 displays the plate temperature profiles along the left insulated/heated plate for each of three channel shapes illustrated in Figure 3. We can see that the heated plate temperature profile in the initial channel exhibits the oscillatory ascending behavior along each heater with the average temperature for each successive heater being greater than for the previous one. Examination of the plate temperature profile for the linear channel case shows that the local plate temperature is everywhere reduced and that the maximum is no longer lies at the channel outlet. For the nonlinear channel, we see that not only have the local plate temperatures decreased everywhere overall, but the maximum temperature for each heater levels off and is also approximately the same. This second result is a bonus for the nonlinear design inasmuch as a uniform



Notes: (a) Initial, (b) optimized linear unheated plate and (c) optimized nonlinear unheated plate

Figure 3.
Channel shapes



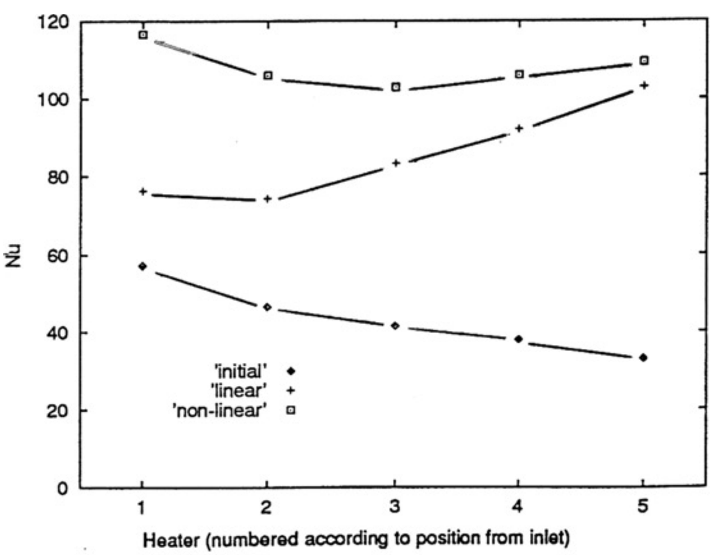
Notes: (a) Initial, (b) optimized linear unheated plate and (c) optimized nonlinear unheated plate

Figure 4.
Temperature variation along the directly heated plate

maximum temperature for all of the heaters was not specified as an objective function, although it could have been. Also, the provision of uniform maximum plate temperatures for the discrete heaters (e.g. computer chips) is a desirable objective from the standpoint of optimizing the operating environment of the chips.

Shown in Figure 5 is the mean Nusselt number Nu for the five discrete heaters in association with the three flat channel shapes explored. For the initial channel, Nu for the first heater is 55, thereafter decreases linearly down to 30 for the fifth heater. We observe that for the third channel with the nonlinear plate, Nu stays approximately in a band between 100 and 115. Both behaviors are in accord with the trend of the heated plate temperature profiles in Figure. 4. In the first heater, Nu = 55 with the initial channel, Nu = 75 with the linear channel and Nu = 115 for the nonlinear channel. In the fifth heater, Nu = 30 with the initial channel, Nu = 100 with the linear channel and Nu = 110 for the nonlinear channel. The enhanced heat transfer coefficients achieved by the two optimized designs relative to the initial design can be explained by realizing that the growth of the momentum boundary layer is retarded by the fluid acceleration due to narrowing of the channel. This action leads to enhancement of the streamwise convective term in the energy equation and a resultant retardation of the development of the thermal boundary layer in the flow region adjacent to the heaters. In the inactive zones between consecutive heaters, the fluid temperature diminishes as the energy diffuses outward with no input from the plate. Then as the next heater is encountered, the plate temperature tends to be lower than if the plate were continuously heated because of the lower fluid temperature near the plate, causing a jump in local heat transfer coefficient.

In sum, it must be emphasized that the most interesting outcome of the present optimization study is the enhancement of the heat transfer coefficient molded by the step-by-step interactive shape optimization of the initial channel design. This goal has been achieved modifying the momentum and energy transfer of the water flow. We summarize the conclusions in the following section.



Notes: (a) Initial, (b) optimized linear unheated plate and (c) optimized nonlinear unheated plate

Figure 5. Mean Nusselt number of the array of discrete heaters

Conclusions

The conclusions drawn from the successful completion of the present study are summarized as follows:

- The output of a CFD code in terms of velocity and temperature fields in a channel provides the input for a shape optimization code. With this linkage, an automatic optimization of the shape of one channel plate is performed in harmony with the design parameters and design constraints.
- The heat transfer coefficients of a cooling channel can be significantly enhanced by optimizing the cross-sectional shape of the channel.
- The shape deformer used in the present study can effectively and efficiently modify channel shapes using only a very few shape-related design variables.

Notes

1. *FLUENT Reference Manual*, available at: www.fluent.com
2. *Sculptor Reference Manual*, available at: www.optimalsolutions.us

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